

(PMAT 401)
M.Sc(Mathematics) P.G Examinations-July 2022
 SEMESTER-IV
MOOCS- NUMERICAL ANALYSIS

TIME: 3Hrs

MaxMarks:60

I Answer any 5 questions out of the 10 short answer questions

5X4=20M

1. Define Absolute, Relative and Percentage Errors.
2. Define Forward difference, Backward difference, Central difference operators
3. Write Newton's Forward Interpolation formula and Backward Interpolation formula.
4. Using Newton-Raphson Method, Find a root of the equation $x^3 - 2\sin x - 2 = 0$ correct up to five decimal places.
5. Define Eigen value and Eigen vector of a matrix.
6. Write Simpson's $\frac{3}{8}$ th formula & Boole's formula.
7. Write the System of Linear equations.
8. Find the third divided difference of the function $f(x) = x^3 + x + 2$ for the arguments 1, 3, 6, 11.
9. Write Second order Runge-Kutta formula.
10. Evaluate $\int_0^1 \frac{dx}{1+x}$ correct to three decimal places by Trapezoidal rule with $h=0.5$.

II Answer all questions.

5X8=40M

11.a) Prove that ① $(1 + \Delta)(1 - \Delta) = 1$

$$\textcircled{2} \mu = \cosh\left(\frac{hD}{2}\right)$$

(OR)

b) Find the absolute, relative and percentage error in x_A when $X_T = 1/7$ and $x_A = 0.1429$.

12.a) Express $f(x) = 15x^4 - 3x^3 - 6x^2 + 11$ in factorial notation.

(OR)

b) State and Prove Newton's divided difference formula.

13. a) The values of x & y are given as below

X	5	6	9	11
f(x)	12	13	14	16

Find the value of y at x=10 by using Lagrange's formulae

(OR)

b) Find $f'(0.6)$ & $f''(0.6)$ from the following table by using Stirling's formula

x	0.4	0.5	0.6	0.7	0.8
f(x)	1.5836	1.7974	2.0442	2.3275	2.6510

14.a) State and prove Newton's Gregory forward Interpolation formula.

(OR)

b) Solve the equations $2x+y+4z=12$, $8x-3y+2z=23$, $4x+11y-z=33$ by Gauss

Elimination method.

15.a) Use Newton Raphson Method to obtain a root, correct to 3 decimal places of the equation $x + \log x = 2$.

(OR)

b) Solve $\frac{dy}{dx} = -2xy^2$ with $y(0) = 1$ and $h=0.2$ on the interval $[0,1]$ using Runge-Kutta

Fourth order method.

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(PMAT 402)
M.Sc(Mathematics) P.G Examinations-July 2022
SEMESTER-IV
INTEGRAL TRANSFORMS

TIME: 3Hrs

MaxMarks:60

SECTION-A

Answer any **FIVE** of the following. Each question carries **4** Marks. **5 x 4 = 20 M**

1. Find the Laplace transform of $[e^{2t} + 4t^3 - 2\sin 2t + 3\cos 3t]$.
2. State and prove First shifting theorem of Laplace transforms.
3. Find the Inverse Laplace transform of $\left[\frac{e^{-2p}}{p^2 - 4p + 20}\right]$ by using Second Shifting theorem.
4. Find the Inverse Laplace transform of $\log\left(\frac{p+3}{p+4}\right)$ by using transform of derivatives.
5. State and prove Modulation theorem.
6. Show that $\int_0^{\infty} \frac{\cos \lambda x}{\lambda^2 + 1} dx = \frac{\pi}{2} e^{-x}; x \geq 0$.
7. Find the finite Fourier cosine transform of $f(x) = \left(1 - \frac{x}{\pi}\right)^2, 0 < x < \pi$.
8. Find the finite Fourier sine and cosine transform of $f(x) = 1$.
9. Define Hankel transform. Find the Hankel transform of $\frac{e^{-ax}}{x}$ taking $xJ_0(px)$ as the Kernel of the transformation.
10. Find the Hankel transform of $\tilde{x}^2 e^{-x}$ by taking $xJ_1(px)$ as the Kernel of the transformation.

SECTION-B

5 x 8 = 40 M

Answer **ALL** questions. Each question carries **8** Marks.

(10a). If $F(t)$ is a function of class Λ and if $L\{F(t)\} = f(p)$ then prove that

$$L\{t^n F(t)\} = (-1)^n \frac{d^n}{dp^n} f(p) \text{ and hence evaluate } L\{t^2 \cos at\}.$$

(OR)

b) State and prove Second shifting theorem of Laplace transforms.

Find $L\{G(t)\}$ where $G(t) = \begin{cases} e^{t-a}, & t > a \\ 0, & t < a \end{cases}$.

(2a). Find the Inverse Laplace transform of $\left\{ \frac{2p+1}{(p+2)^2(p-1)^2} \right\}$ by using partial fractions.

(OR)

b) State and prove Convolution theorem.

(3a). Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$

and i) $\int_{-\infty}^{\infty} \frac{\sin p a \cos p x}{p} dp$ ii) $\int_0^{\infty} \frac{\sin p}{p} dp$.

(OR)

b) State and prove Parseval's Identity for Fourier Transforms.

Evaluate $\int_0^{\infty} \frac{x^2}{(a^2+x^2)^2} dx$ ($a > 0$) by using Parseval's Identity.

(4a) Find the Finite Fourier sine and cosine transform of $f(x)$ if

$$f(x) = \begin{cases} 1, & 0 < x < \frac{\pi}{2} \\ -1, & \frac{\pi}{2} < x < \pi \end{cases}$$

(OR)

b) Find the Finite Fourier sine and cosine transform of $f(x) = x^3, 0 < x < \pi$.

(5a) Find the Hankel transform of $\frac{d^2 f}{dx^2} + \frac{1}{x} \frac{df}{dx}$ when p is the root of the equation

$$J_0(ap) = 0.$$

(OR)

b) Find the Hankel transform of $f(x) = \begin{cases} a^2 - x^2 & \text{if } 0 < x < a; n = 0 \\ 0, & \text{if } x > a; n = 0 \end{cases}$.

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(PMAT 403)
M.Sc(Mathematics) P.G Examinations-July 2022
SEMESTER-IV
LINEAR PROGRAMMING

TIME: 3Hrs

MaxMarks:60

Section A

Answer any 5 out of Ten short answer questions

$5 \times 4 = 20$

1. Explain Models in O.R.
2. Explain slack variables, surplus variables, feasible solution, Non degenerate, optimum solution.
3. Explain Degeneracy and Degeneracy in LPP.
4. Define unbounded and Infeasible solution.
5. State Dual-Primal Relationship
6. Write Dual simplex Method.
7. State the Transportation problem in LPP, define transportation problem.
8. Explain Least cost Method
9. Describe Alogrtim for Assisnment problem
10. Explain Hungarian Method.

Section B

Answer the following Questions

5 × 8 = 40

(1) a) Solve Using LPP by Simplex Method

$$\text{Max } Z = 3x_1 + 5x_2 + 4x_3,$$

$$2x_1 + 3x_2 \leq 8, 3x_1 + 2x_2 + 4x_3 \leq 15, 2x_2 + 5x_3 \leq 10.$$

OR

b) Solve Using LPP by Simplex Method

$$\text{Max } Z = 3x_1 + 2x_2, x_1 + x_2 \leq 4, x_1 - x_2 \leq 2.$$

(2) a) Use Two phase method and solve

$$\text{Min } Z = x_1 - 2x_2 - 3x_3, -2x_1 + x_2 + 3x_3 = 2, 2x_1 + 3x_2 + 4x_3 = 1.$$

OR

b) Solve by using Big-M method

$$\text{Max } Z = -2x_1 - x_2, 3x_1 + x_2 = 3, 4x_1 + 3x_2 \geq 6, x_1 + 2x_2 \leq 4.$$

(3) a) State and prove Fundamental theorem of Duality.

OR

b) Use Dual simplex Method to solve

$$\text{Min } Z = 3x_1 + x_2, x_1 + x_2 \geq 1, 2x_1 + 3x_2 \geq 2$$

14 a) Solve Using VAM Method

	M_1	M_2	M_3	M_4	supply
A	6	3	5	7	22
B	5	9	2	7	15
C	5	7	8	6	8
Demand	7	12	17	9	

OR

b) Solve Using North west corner Method

	D_1	D_2	D_3	D_4	supply
A	6	4	1	5	14
B	8	9	2	7	16
C	4	3	6	2	5
Demand	6	10	15	4	

15 a) Solve by Hungarian Method

	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

OR

b) Solve Assignment problem

	T_1	T_2	T_3	T_4	T_5
S_1	26	14	10	12	9
S_2	31	27	30	14	16
S_3	15	18	16	25	30
S_4	17	12	21	30	25
S_5	20	19	25	16	10

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(PMAT 404)
M.Sc(Mathematics) P.G Examinations-July 2022
SEMESTER-IV
FUNCTIONAL ANALYSIS

TIME: 3 Hrs

Max Marks:60

SECTION-A

Answer any **FIVE** of the following. Each question carries 4 Marks. $5 \times 4 = 20 \text{ M}$

1. Define Normed space and Banach space.
2. Prove that a compact subset M of a metric space is closed and bounded.
3. Define Inner Product Space and Hilbert Space.
4. State and prove Schwarz Inequality.
5. Define Hilbert-Adjoint operator.
6. Define Self-Adjoint, Unitary and Normal Operators.
7. Prove that every Hilbert space is reflexive.
8. Define Merger set. State Baire's category theorem in complete metric space.
9. Show that a contraction mapping T on a metric space X is a continuous mapping.
10. Define Fredholm and Volterra integral equations.

SECTION-B

Answer **ALL** questions. Each question carries 8 Marks. $5 \times 8 = 40 \text{ M}$

- (a). Prove that a subspace Y of a Banach space X is complete if and only if the set Y is closed in X .

(OR)

b) State and prove Riesz's Lemma.

12 a). Let Y be any closed subspace of a Hilbert space H , then prove that

$$H = Y \oplus Y^\perp, \text{ where } Z = Y^\perp.$$

(OR)

b) State and prove Bessel's Inequality.

13 a) State and prove Riesz Representation Theorem.

(OR)

b). Let the operators $U: H \rightarrow H$ and $V: H \rightarrow H$ be unitary operators and H is a Hilbert space. Then prove that a bounded linear operator T on a complex Hilbert space H is unitary if and only if T is isometric and surjective.

14 a). State and prove Generalized Hahn-Banach Theorem.

(OR)

b). State and prove Open Mapping Theorem.

15 a). State and prove Banach fixed point theorem.

(OR)

b) Explain the applications of Banach's theorem to Linear equations.

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(PMAT 405)
M.Sc(Mathematics) P.G Examinations-July 2022
SEMESTER-IV
MEASURE AND INTEGRATION

TIME: 3 Hrs

Max Marks:60

Answer any 5 questions out of 10 short answer questions

5 X 4 = 20M

1. Let m be a countably additive measure defined for all sets in a σ -algebra \mathcal{M} . Let $\{E_n\}$ be any sequence of sets in \mathcal{M} . Then $m(\cup E_n) \leq \sum m(E_n)$
2. Let $E \subset [0,1)$ be a measurable set. Show that for each $y \in [0,1)$ the set $E+y$ is measurable and $m(E+y) = m(E)$
3. Construct a sequence of $\{f_n\}$ of nonnegative, Riemann integrable functions such that f_n increases monotonically to f . Comment on the changing of order of integration and the limiting process.
4. State and prove Lebesgue Convergence Theorem
5. Let f be defined by $f(0) = 0$ and $f(x) = x^2 \sin(1/x^2)$ for $x \neq 0$. Is f of bounded variation on $[-1,1]$
6. Show that the sum of two absolutely continuous functions is also absolutely continuous
7. Define a signed measure
8. Show that if μ and ν are measures on a σ -algebra. Then $\mu + \nu$ is also a measure
9. Define an outer measure
10. State Tonelli's Theorem

Answer the following Questions

5X8 = 40 Marks

- 11 (a) Let $\{E_i\}$ be a sequence of measurable sets with Lebesgue measure μ . Then show that $\mu(\cup E_i) \leq \sum \mu(E_i)$ and if the sets E_i are pair wise disjoint, then $\mu(\cup E_i) = \sum \mu(E_i)$.

(OR)

- (b) Let $\{E_i\}$ be an infinite decreasing sequence of measurable sets, that is, a sequence with $E_{i+1} \subset E_i$ for each i . Show that if $\mu(E_1) < \infty$. Then $\mu(\cap_{i=1}^{\infty} E_i) = \lim_{n \rightarrow \infty} \mu(E_n)$

- 12 (a) Define integral of a nonnegative measurable function. State and prove the Monotone Convergence Theorem

(OR)

- (b) Let f be a nonnegative function which is integrable over a set E . Then given $\varepsilon > 0$ there is a $\delta > 0$ such that for every set $A \subset E$ with $m(A) < \delta$ we have $\int_A f < \varepsilon$.

- 13 (a) If f is bounded and measurable on $[a, b]$ and $F(x) = \int_a^x f(t) dt + F(a)$ then show that $F'(x) = f(x)$ for almost all x in $[a, b]$

(OR)

- (b) If f is absolutely continuous on $[a, b]$ and $f'(x) = 0$ almost everywhere, then show that f is constant

- 14 (a) Let ν be signed measure and E be a measurable set such that $0 < \nu(E) < \infty$. Show that there is a positive set A contained in E with $\nu(A) > 0$

(OR)

- (b) State and prove the Radon Nikodym Theorem

- 15 (a) Let \mathcal{B} be an algebra. If $A \in \mathcal{B}$ and $\{A_i\}$ is any sequence of sets in \mathcal{B} such that $A \subset \cup_{i=1}^{\infty} A_i$, then $\mu(A) \leq \sum_{i=1}^{\infty} \mu(A_i)$

(OR)

- (b) State and prove Fubini's Theorem

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